Homework 4:

Let's address each of your questions using the set of integers \( S = \{0, 1, 2, 3, 4, 5, 6\} \):

1. \*\*4 + 4\*\*: In standard arithmetic, 4 + 4 equals 8. However, if we are considering operations within the given set \( S \) and are assuming standard modulo arithmetic (where the results are taken modulo 7, since 7 is the largest number not in the set), then the result of 4 + 4 would be 1 (since 8 mod 7 equals 1).

2. \*\*3 x 5\*\*: Similarly, 3 multiplied by 5 equals 15 in standard arithmetic. Under the assumption of modulo 7 arithmetic, 15 mod 7 equals 1.

3. \*\*Inverse of 3 under:\*\*

- \*\*Addition\*\*: The additive inverse of an element \( a \) in a set is an element \( b \) such that \( a + b \) equals the identity element under addition, which is 0. In modulo 7 arithmetic, the additive inverse of 3 is 4, because \( 3 + 4 = 7 \), and 7 mod 7 equals 0.

- \*\*Multiplication\*\*: The multiplicative inverse of an element \( a \) in a set is an element \( b \) such that \( a \times b \) equals the identity element under multiplication, which is 1. In modulo 7 arithmetic, the multiplicative inverse of 3 is 5, because \( 3 \times 5 = 15 \), and 15 mod 7 equals 1.

To clarify, these answers are based on the assumption that the arithmetic operations are performed modulo 7, as the set \( S \) consists of integers from 0 to 6.

Let's address each of your questions:

### 1. Group Properties of \( S = \{0, 1, 2, 3, 4, 5, 6\} \) under Addition

To determine if \( S \) with the operation '+' forms a group, we need to check several properties:

1. \*\*Closure\*\*: Does the operation '+' result in an element that is also in \( S \) for all pairs of elements in \( S \)?

2. \*\*Associativity\*\*: Is the operation '+' associative?

3. \*\*Identity Element\*\*: Is there an identity element in \( S \) such that \( a + e = e + a = a \) for all \( a \) in \( S \)?

4. \*\*Inverse Element\*\*: Does every element in \( S \) have an inverse in \( S \) such that \( a + b = b + a = e \), where \( e \) is the identity element?

Given \( S \) under standard addition, we notice that the operation '+' is not closed in \( S \). For example, \( 4 + 4 = 8 \), which is not in \( S \). Therefore, \( S \) under the operation '+' does not form a group.

### 2. Computing \(-13 \mod 5\)

The modulo operation finds the remainder when one number is divided by another. For \(-13 \mod 5\):

-13 mod 5 = -3

### 3. Polynomial Analysis

Given the polynomial \( x^3 - x^2 + 4x - 12 \):

- \*\*Finding a Positive Root\*\*: One way to find a root is to try simple integers. Let's try 1, 2, 3, etc., and see if any of these satisfy the polynomial equation.

- \*\*Degree of the Polynomial\*\*: The degree of a polynomial is the highest power of the variable in the polynomial. In this case, the degree is 3, as the highest power of \( x \) is \( x^3 \).

Let's calculate a positive root for your polynomial.

The positive root of the polynomial \( x^3 - x^2 + 4x - 12 \) is \( x = 2 \). This means that when \( x = 2 \), the polynomial equals zero.

To summarize:

- \( S = \{0, 1, 2, 3, 4, 5, 6\} \) with the operation '+' does not form a group under standard addition.

- \(-13 \mod 5 = -3\).

- The positive root of the polynomial \( x^3 - x^2 + 4x - 12 \) is 2.

- The degree of the polynomial is 3.